Trade Liberalization, Educational Choice, and Income Distribution

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Abstract

Recent empirical studies reveal that trade liberalization affects people's educational attainment differently for different skill groups. This paper constructs an overlapping generations model with heterogeneity in agents' ability to examine the dynamic impacts of trade liberalization on individuals' educational choices and income distribution. Our focus is on a developing economy that has a comparative advantage in the low-skill-intensive, agricultural sector, since it is far from obvious and important to know whether opening to trade induces the development of industries that serve as an engine of growth and give incentives to citizens to acquire human capital in those countries. Our theoretical model illustrates that in the short run, an export expansion of the agricultural sector will increase the low-skilled wage relative to the high-skilled, which discourages individuals from getting an education. In the long run, however, trade induces capital accumulation, raising the wage rate for high-skilled workers who engage in capital-intensive manufacturing industries. As a result, "education polarization" arises such that more individuals will receive tertiary education, despite that the pool of low-skilled workers also expands as a direct consequence of the expansion of the agricultural sector. We also illustrate transitional dynamics that follow trade liberalization and examine the impact of trade on different skill groups in different generations.

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1 Introduction

Globalization has benefited firms, helping them build their supply chains beyond the national border and expand their sales over the world. Consumers also benefit from globalization as they became able to consume a wider variety of products at lower prices. But it has also been argued that it has widened income inequality within countries, especially in developed countries. The impacts on developing countries are relatively less documented. The Stolper-Samuelson theorem implies that trade liberalization disproportionately benefits low-skilled workers that intensively engage in their traditional export-goods sectors, thereby reducing income inequality. Data do not necessarily show such evidence, however. Goldberg and Pavcnik (2007), among others, report that globalization entailed the expansion of income inequality in many developing countries.

This paper investigates the effect of trade liberalization on educational choice and income inequality in developing countries. There are many possible reasons why globalization entails the expansion of income inequality in developing countries. Feenstra and Hanson (1996) attribute it to developed countries' outsourcing their least-skill-intensive goods, which are skill-intensive from the viewpoint of developing countries. Zhu and Trefler (2005) argue that Southern technological catch-up shifted skill-intensive sectors to some developing countries, which in turn widened income inequality there. Blanchard and Olney (2017) point out the possibility that an increase in the export of less-skill-intensive goods increased the pool of low-skilled workers through educational choice and hence mitigated the Stolper-Samuelson effect.

In this paper, we emphasize the dynamic effect of trade liberalization. An induced expansion of low-skill-intensive export sectors, such as agricultural and low-skilled manufactured goods, narrows income inequality in the short run. As savings by those low-skill labor increase, however, capital will gradually accumulate, which raises the wage rate for high-skill labor that is intensively used in the capital-intensive sectors. Thus, in the long run, income inequality may expand as a result of trade liberalization. We also examine how such dynamic impacts of trade liberalization affect individuals' choices of schooling and how these effects interact with each other and affect income inequality across different generations within developing countries.

To analyze the dynamic effects of trade liberalization on income distribution, we build a T-period overlapping generations (OLG) trade model, where individuals, heterogeneous in their ability, choose the length of schooling: working immediately after primary education (which corresponds to minimum schooling in our framework because we take primary education compulsory), completing secondary education, or completing tertiary education. Those who have received only primary education (i.e., minimum schooling) are categorized as low-skilled and work in the low-skill-intensive sector, which we call the agricultural sector. Those who have completed secondary education but not tertiary education are considered middle-skilled and work in the manufacturing sector. Finally, individuals who completed tertiary education are called high-skilled and work in the manufacturing sector with productivity proportionately higher than middle-skilled workers. Trade liberalization affects these three types of workers differently and their differential effects differ between the short run and the long run. Individuals choose their own education levels in anticipation of the dynamic wage effect of trade liberalization.

Instead of looking at the impact of trade liberalization on low-skilled and high-skilled workers, we divide workers into the above three types and examine how the impacts are different across different groups. It has been documented that in many developed countries, employment and wages of middle-wage occupations declined, while those of low-wage and high-wage grew (e.g., Acemoglu and Autor, 2011). The effects of trade on the labor market are also too complicated, in practice, to be captured by a model with two types of workers. Keller and Utar (2016) find evidence that in Denmark import competition caused job polarization with an employment shift from the middle-wage manufacturing. Some studies, such as Egger and Kreickemeier (2012), Kohl (2020), and Furusawa et al. (2020), employ the Melitz (2003)-type intra-industry trade models and analyze income inequalities both between low-skilled and high-skilled and among high-skilled workers. Blanchard and Willmann (2016) analyze the inequality among skilled workers in a Ricardian model with a continuum of goods, developed by Dornbusch et al. (1977). It is important to incorporate more than two types of workers in the model to derive rich effects on the labor market observed in reality.

The study on the effect of trade on education and income distribution dates back to the 1980s. Findlay and Kierzkowski (1983) conduct a seminal study on this subject in an inter-industry trade model.[1] Borsook (1987) extends their model to the one with heterogeneous agents in their abilities and shows that workers with higher abilities utilize more educational capital, which generates lifetime income inequality. Falvey et al. (2010) also analyze an educational-choice model, paying particular attention to the announcement effect of trade liberalization on lifetime income distribution between young and old workers. Auer (2015) shows in a similar context that rich countries reap dynamic gains from trade more than poor countries through human capital accumulation and thus trade leads to cross-country divergence in income and welfare in the long run.

Harris and Robertson (2013) examine the dynamic impact of trade liberalization in a model with physical-capital as well as human-capital accumulation. Their model is in the spirit of Findlay and Kierzkowski (1983) and Borsook (1987) and closely related to ours. They show numerically that trade liberalization entails the long-run accumulation of skilled labor and physical capital as well as the immediate Stolper-Samuelson effect on the skill wage premium. We share the aim of the analysis with them and derive some similar results. Unlike their infinitely-lived, representative-agent model, however, we adopt an OLG model, which

[1]Some studies analyze the effect of trade on education, and income distribution for credit-constrained economies. Trade liberalization in skill-scarce countries lowers educational costs, increases wages of low-skilled workers, and may promote educational attainment by mitigating credit constraints. See Cartiglia (1997), Ranjan (2001, 2003), and Chesnokova and Krishna (2009).

we believe fits better to our goal. First, we can model more realistic educational choices, such as receiving education at an early stage of life. This is important when we assess the impact of trade liberalization on different generations because those impacts are naturally different depending on which life stage they are in when trade is liberalized. With an OLG model, we can also discuss intergenerational income redistribution caused by trade liberalization. Another important difference between our model and theirs is that we introduce a group of middle-skilled workers as well as those of low-skilled and high-skilled workers. With three skill groups, we can analyze a richer impact of trade on the labor market and show that trade liberalization in developing countries entails wage polarization in the long run. We also discuss the resulting lifetime-income inequality among different skill groups and different generations. Similarly to our paper, Danziger (2017) examines the dynamic effect of trade liberalization on workers with different abilities and ages. However, he adopts an intra-industry trade model that fits better to the analysis for developed countries.

In an OLG model with endogenous educational choice and capital accumulation, we show that (i) the export expansion in the agricultural goods induced by trade liberalization raises the wage rate for low-skill labor relative to middle/high-skill labor (the Stolper-Samuelson effect), hence narrowing lifetime income inequality between low- and middle-skilled workers and reducing the pool of individuals who receive more than primary education and; (ii) the export expansion leads to capital accumulation, raises the wage rate of middle/high-skill labor relative to the educational cost, and lowers the rental rate (or equivalently the interest rate), and hence increases the return to education and lifetime income inequality between middle- and high-skilled workers in the long run; and (iii) income inequality between middle- and high-skilled workers increases with capital accumulation during transition induced by the export expansion (shown numerically in Section 3). Critical to the novel results in (ii) and (iii) is the capital accumulation induced by the expansion of agricultural exports. The export expansion increases capital demand immediately, but capital supply increases only gradually as low-skilled workers, who benefit from trade liberalization, increase their savings. After an immediate hike upon the trade liberalization, the rental rate gradually declines as capital accumulates, which in turn increases the wage rate for middle/high-skill labor and hence the returns to education.

The organization of this paper is as follows. Section 2 constructs a dynamic model with educational choice, and presents a theoretical analysis of trade liberalization in the steady state. Section 3 conducts a numerical analysis of the dynamic transition path from the closed-economy steady state after the country opens to trade, and derives rich effects of trade liberalization on factor prices, physical- and human-capital accumulations, individuals' educational choices, and income inequality. Section 4 concludes the paper.

2 The Model Analysis

2.1 The Model Environment and Equilibrium Conditions

We consider an OLG model with discrete time. Individuals in generation τ are "born" at $t = \tau$ and live for T periods; we assume all individuals receive primary education so that τ should be considered as the time period immediately after individuals completed their primary education. Within cohorts, individuals differ from each other in terms of their ability, denoted by a. The ability a is a random variable that follows a probability distribution function of F, which is differentiable and common across generations. At time $t = \tau$, each individual in generation τ is born with a given ability and chooses whether or not to proceed to secondary education. If she does not choose to proceed to secondary education, she immediately starts working and supplies one unit of low-skill labor in each period. If she decides to proceed to secondary education, she will make a further decision as to whether to receive tertiary education when she completes secondary education. We call those who have received secondary but not tertiary education middle-skilled workers and those who have received tertiary education high-skilled workers. Both middle-skilled and high-skilled workers supply high-skill labor in each period. Those who have received tertiary education though provide greater units of high-skill labor than those who have the same ability but have not received tertiary education, as we shortly describe in more detail.

There are two goods, which we call the agricultural good, denoted by n, and the manufactured good, denoted by m. The agricultural good is for pure consumption while the manufactured good is for capital usage as well as for final consumption. We also explicitly introduce the education sector, denoted by e, that provides secondary and tertiary education for students. Capital and low-skill labor are used to produce agricultural goods, while capital and high-skill labor are required to produce manufactured goods and educational services. We let $p_{n,t}$, $p_{m,t}$, and $p_{e,t}$ denote the prices of agricultural and manufactured goods and educational services, respectively, in period t and choose the manufactured good as a numéraire (i.e., $p_{m,t} = 1$). All markets are perfectly competitive.

The unit cost functions of the goods and services, χ_i, $i = n, m, e$, are specified as

$$\chi_n(r_t, w_{l,t}) = r_t^{\alpha_n} w_{l,t}^{1-\alpha_n}, \tag{1a}$$

$$\chi_i(r_t, w_{h,t}) = r_t^{\alpha_i} w_{h,t}^{1-\alpha_i}, \quad i = m, e, \tag{1b}$$

where r_t , $w_{l,t}$, and $w_{h,t}$ are the rental rate, the wage rate for low-skill labor, and that for high-skill labor, respectively, while α_i, for $i = n, m, e$, is a Cobb-Douglas share parameter. We assume, for expositional simplicity, that $\alpha_n \leq \alpha_m$.[2]

There is a continuum of individuals of a unit mass with heterogeneous ability a in each generation. The lifetime utility of individuals (a, τ) with the rate of time preferences ρ is

[2]This assumption is natural, especially in developing countries where the production of agricultural goods largely relies on manual labor rather than machines.

given by

$$u = \sum_{t=\tau}^{\tau+T-1} \left(\frac{1}{1+\rho}\right)^{t-\tau} \left[\beta \ln c_{n,t}(a,\tau) + (1-\beta) \ln c_{m,t}(a,\tau)\right]; \; \rho > 0, \; 0 < \beta < 1, \tag{2}$$

where $c_{n,t}$ and $c_{m,t}$ are consumption of agricultural and manufactured goods, respectively. Each individual maximizes her lifetime utility subject to the budget constraints for $t = \tau, \tau+1, ..., \tau+T-1$:

$$\begin{aligned} k_{t+1}(a,\tau) &= (1+r_t)k_t(a,\tau) + w_t(a,\tau) - c_t(a,\tau), \\ c_t(a,\tau) &\equiv p_{n,t}c_{n,t}(a,\tau) + c_{m,t}(a,\tau), \end{aligned} \tag{3}$$

where k_t is her capital holding and c_t her expenditure on consumption, while w_t is the wage rate if she works at time t, while it represents the educational cost (so that w_t takes a negative value) if she is in school. Individuals are born with no capital. The non-Ponzi game condition implies that individuals die without debt, i.e., $k_{\tau+T}(a,\tau) \geq 0$.

Generation-τ individuals who decide not to receive secondary education start working as low-skilled workers immediately after their birth at time τ, receiving $w_t = w_{l,t}$ in each period t regardless of their ability. Individuals who only receive up to secondary education go to school for θ_m periods before they start providing high-skill labor as middle-skilled workers. The education enables each of them with her ability of a to provide a units of high-skill labor in each period, receiving $w_t = w_{h,t}a$ as a reward. Individuals who receive tertiary education spend their first $\theta_h(> \theta_m)$ periods in school. After that, each of them with her ability of a provides ha (where $h > 1$) units of high-skill labor as a high-skilled worker. She earns $w_{h,t}ha$ in each period t for the rest of her life. We assume for simplicity that education fees in each period are the same between secondary and tertiary education and that they are given by $w_t = -\gamma p_{e,t}$, where γ represents the reciprocal of the education productivity. We assume $T - \theta_m < h(T - \theta_h)$ since otherwise, no one has an incentive to receive tertiary education.

Each individual (a, τ) chooses an optimal consumption stream over her lifetime. We define the compound interest rate from time t to s and the compound discount factor from time t onward by

$$R(t,s) \equiv \begin{cases} 1 & \text{if } \; s = t, \\ \prod_{u=t+1}^{s}(1+r_u) & \text{if } \; s \geq t+1, \end{cases}$$

and

$$\Gamma_t(\tau) \equiv \sum_{s=t}^{\tau+T-1} \left(\frac{1}{1+\rho}\right)^{s-t} = \frac{1-\left(\frac{1}{1+\rho}\right)^{\tau+T-t}}{1-\frac{1}{1+\rho}},$$

respectively. Then, the individual's lifetime income since time t can be written as

$$I_t(a,\tau) \equiv (1+r_t)k_t(a,\tau) + \sum_{s=t}^{\tau+T-1} \frac{w_s(a,\tau)}{R(t,s)}. \tag{4}$$

Her optimal expenditure at time $s = t, t+1, \cdots, \tau+T-1$ is given by

$$c_s(a,\tau) = \frac{R(t,s)}{(1+\rho)^{s-t}} \frac{I_t(a,\tau)}{\Gamma_t(\tau)}. \tag{5}$$

In particular, her expenditure plan at her birth is given by

$$c_t(a,\tau) = \frac{R(\tau,t)}{(1+\rho)^{t-\tau}} \frac{I_\tau(a,\tau)}{\Gamma_\tau(\tau)}, \text{ for } t = \tau, \tau+1, \cdots, \tau+T-1, \tag{6}$$

and consequently, her optimal consumption levels of agricultural and manufactured goods are given by

$$\begin{aligned} c_{n,t}(a,\tau) &= \frac{\beta c_t(a,\tau)}{p_{n,t}}, \\ c_{m,t}(a,\tau) &= (1-\beta)c_t(a,\tau). \end{aligned} \tag{7}$$

Once the income and expenditure streams are given, the optimal capital holding at any time t

is determined as

$$k_t(a,\tau) = R(\tau, t-1) \sum_{s=\tau}^{t-1} \left[\frac{w_s(a,\tau)}{R(\tau,s)} - \frac{c_s(a,\tau)}{R(\tau,s)} \right]. \tag{8}$$

Individuals choose their education to maximize their lifetime income. We assume for simplicity that once they start working, they do not go back to school.[3] Under the assumption of perfect foresight, no one has an incentive to change her educational plan made at her birth, unless an unexpected event, such as the trade liberalization that we consider here, arises thereby changing her future income stream. Our simplifying assumption above implies that at the time of trade liberalization, only students then would change their educational plan. Lifetime incomes at birth for low-skilled, middle-skilled, and high-skilled workers are given by

$$I_l(\tau) \equiv W_l(\tau) \equiv \sum_{t=\tau}^{\tau+T-1} \frac{w_{l,t}}{R(\tau,t)}, \tag{9a}$$

$$I_m(a,\tau) \equiv -P_{em}(\tau) + W_m(\tau)a, \tag{9b}$$

$$I_h(a,\tau) \equiv -P_{eh}(\tau) + W_h(\tau)ha. \tag{9c}$$

where

$$P_{ei}(\tau) \equiv \sum_{t=\tau}^{\tau+\theta_i-1} \frac{\gamma p_{e,t}}{R(\tau,t)},\ i = m,h;\ \ W_i(\tau) \equiv \sum_{t=\tau+\theta_i}^{\tau+T-1} \frac{w_{h,t}}{R(\tau,t)},\ i = m,h.$$

An individual (a,τ) chooses her education so as to maximize her lifetime income at birth, denoted by $I(a,\tau)$, i.e., $I(a,\tau) = \max\{I_l(\tau), I_m(a,\tau), I_h(a,\tau)\}$.

As expected, individuals with the lowest abilities do not go to school, those with medium abilities receive only up to the secondary education to become middle-skilled workers, and those with the highest abilities complete the tertiary education to become high-skilled workers. Let $a_l(\tau)$ and $a_h(\tau)$ denote the ability thresholds that separate the low-skilled and middle-

[3]It can be shown that unless the educational costs, relative to its benefits, drastically decline in the future, individuals complete their education at the beginning of their lives. It is because postponing education would reduce the length of time during which they reap benefits from the resulting higher wages.

skilled workers and between middle-skilled and high-skilled workers in generation τ. We focus on the empirically-plausible case that all those three types of individuals exist so that $0 < a_l(\tau) < a_h(\tau) < \infty$. The thresholds $a_l(\tau)$ and $a_h(\tau)$ satisfy $I_l(\tau) = I_m(a_l(\tau), \tau)$ and $I_m(a_h(\tau), \tau) = I_h(a_h(\tau), \tau)$, respectively, so we have from (9) that

$$a_l(\tau) = \frac{P_{em}(\tau) + W_l(\tau)}{W_m}, \tag{10}$$

$$a_h(\tau) = \frac{P_{eh}(\tau) - P_{em}(\tau)}{W_h(\tau) - W_m(\tau)}. \tag{11}$$

Having characterized individuals' choices, we now derive key aggregate variables to obtain the equilibrium conditions. Aggregate expenditure at time t is given by

$$C_t = \sum_{\tau=t-T+1}^{t} \int_0^{\infty} c_t(a, \tau)\, dF(a), \tag{12}$$

while aggregate capital is the sum of all individual savings:

$$K_t = \sum_{\tau=t-T+1}^{t} \int_0^{\infty} k_t(a, \tau)\, dF(a). \tag{13}$$

Individuals' educational choices determine labor supplies and the mass of students. The low-skill labor supply equals the mass of low-skilled workers:

$$L_t = \sum_{\tau=t-T+1}^{t} F\big(a_l(\tau)\big). \tag{14}$$

The high-skill labor supply is the total labor supplied by middle-skilled and high-skilled workers:

$$H_t = \sum_{\tau=t-T+1}^{t-\theta_m} \int_{a_l(\tau)}^{a_h(\tau)} a\, dF(a) + h \sum_{\tau=t-T+1}^{t-\theta_h} \int_{a_h(\tau)}^{\infty} a\, dF(a). \tag{15}$$

The mass of student S is the sum of all individuals who enroll in secondary and tertiary education:

$$S_t = \sum_{\tau=t-\theta_m+1}^{t} \left[F\left(a_h(\tau)\right) - F\left(a_l(\tau)\right)\right] + \sum_{\tau=t-\theta_h+1}^{t} \left[1 - F\left(a_h(\tau)\right)\right]. \tag{16}$$

Now, we are ready to present the conditions that characterize the equilibrium. The first is the zero-profit conditions in all sectors at any t, i.e., $p_{i,t} = \chi_{i,t}$ for $i = n, m, e$. These conditions, together with (1), allow us to express wage rates, $w_{l,t}$ and $w_{h,t}$, and the price of educational service $p_{e,t}$ as functions of the rental rate r_t and the price of agricultural goods $p_{n,t}$:

$$w_l(r_t, p_{n,t}) = \left(\frac{p_{n,t}}{r_t^{\alpha_n}}\right)^{1/(1-\alpha_n)}, \tag{17a}$$

$$w_h(r_t) = r_t^{-\alpha_m/(1-\alpha_m)}, \tag{17b}$$

$$p_{e,t}(r_t) = r_t^{\alpha_e} w_h(r_t)^{1-\alpha_e} = r_t^{(\alpha_e-\alpha_m)/(1-\alpha_m)}. \tag{17c}$$

The education fee $p_{e,t}$ increases with r_t if and only if $\alpha_e > \alpha_m$; an inrease in r_t directly raises $p_{e,t}$ while it indirectly lowers $p_{e,t}$ through a decrease in $w_{h,t}$, the effect of which is small if α_m is small. The closed-economy, market-clearing conditions for goods and educational services are given by

$$Y_{n,t} = \beta C_t / p_{n,t}, \tag{18a}$$

$$Y_{m,t} = (1-\beta)C_t + (K_{t+1} - K_t), \tag{18b}$$

$$Y_{e,t} = \gamma S_t, \tag{18c}$$

where $Y_{i,t}$ denotes the aggregate supply of good/service i. The full employment conditions for capital, low-skill labor, and high-skill labor are expressed as

$$K_t = \frac{\partial \chi_n}{\partial r}(r_t, w_{l,t})Y_{n,t} + \frac{\partial \chi_m}{\partial r}(r_t, w_{h,t})Y_{m,t} + \frac{\partial \chi_e}{\partial r}(r_t, w_{h,t})Y_{e,t}, \tag{19a}$$

$$L_t = \frac{\partial \chi_n}{\partial w}(r_t, w_{l,t})Y_{n,t}, \tag{19b}$$

$$H_t = \frac{\partial \chi_m}{\partial w}(r_t, w_{h,t})Y_{m,t} + \frac{\partial \chi_e}{\partial w}(r_t, w_{h,t})Y_{e,t}, \tag{19c}$$

where we have used Shephard's lemma that unit factor demands are given by the derivatives of the unit cost functions.

We define the wage rates for the low-skill labor and high-skill labor both relative to the rental rate by

$$\omega_l(r, r_t p_{n,t}) \equiv \frac{w_l(r_t, p_{n,t})}{r_t} = \left(\frac{p_{n,t}}{r_t}\right)^{1/(1-\alpha_n)}, \quad \omega_h(r_t) \equiv \frac{w_h(r_t)}{r_t} = r_t^{-1/(1-\alpha_m)}.$$

Then, the equilibrium conditions, (18) and (19), can be reduced to the market clearing conditions for capital and the agricultural goods, which characterize the equilibrium time paths of the two endogenous variables $\{r_t, p_{n,t}\}$:

$$K_t = \frac{\alpha_n}{1-\alpha_n}\omega_l(r_t, p_{n,t})L_t + \frac{\alpha_m}{1-\alpha_m}\omega_h(r_t)H_t + \frac{\alpha_e - \alpha_m}{1-\alpha_m}\omega_h(r_t)^{1-\alpha_e}\gamma S_t, \tag{20a}$$

$$\frac{\beta C_t}{p_{n,t}} = \frac{\omega_l(r_t, p_{n,t})^{\alpha_n} L_t}{1-\alpha_n}, \tag{20b}$$

where the capital-market clearing condition (20a) is derived by substituting (18c), (19b), and (19c) into (19a), while the agricultural-good market-clearing condition (20b) is derived by substituting (19b) into (18a).

2.2 The Closed-Economy Steady-State Equilibrium

The dynamic equilibrium of the closed economy is presented by a path $\{r_t, p_{n,t}\}$ that satisfies (20) and the accompanying paths of other variables derived from (1), (7), (8), (11), and (17). Here, we characterize the closed-economy steady-state equilibrium, to be compared with the free-trade steady-state derived in the next section. The closed-economy steady-state equilibrium is obtained from the time-invariant paths of the endogenous variables (denoted without the time subscript) determined by $\{r_t, p_{n,t}\} = \{r, p_n\}$ that satisfies the equilibrium equations in (20).

We write the steady-state lifetime income as $\bar{I}_l(r, p_n)$, $\bar{I}_m(a, r)$, and $\bar{I}_h(a, r)$, respectively for the three types of workers, where the functions, $\bar{I}_l$, $\bar{I}_m$, and $\bar{I}_h$, are defined by

$$\bar{I}_l(r, p_n) \equiv w_l(r, p_n) \sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^{t-1}, \tag{21a}$$

$$\bar{I}_m(a, r) \equiv -\gamma p_e(r) \sum_{t=1}^{\theta_m} \left(\frac{1}{1+r}\right)^{t-1} + w_h(r) a \sum_{t=\theta_m+1}^{T} \left(\frac{1}{1+r}\right)^{t-1}, \tag{21b}$$

$$\bar{I}_h(a, r) \equiv -\gamma p_e(r) \sum_{t=1}^{\theta_h} \left(\frac{1}{1+r}\right)^{t-1} + w_h(r) h a \sum_{t=\theta_h+1}^{T} \left(\frac{1}{1+r}\right)^{t-1}. \tag{21c}$$

It follows from $\bar{I}_m(a_h, r) = \bar{I}_h(a_h, r)$ that the threshold ability for tertiary education is given by

$$\bar{a}_h(r) = D_h(r) \frac{\gamma p_e(r)}{w_h(r)}, \tag{11'}$$

where

$$D_h(r) \equiv \frac{\sum_{t=\theta_m+1}^{\theta_h} \left(\frac{1}{1+r}\right)^{t-1}}{h \sum_{t=\theta_h+1}^{T} \left(\frac{1}{1+r}\right)^{t-1} - \sum_{t=\theta_m+1}^{T} \left(\frac{1}{1+r}\right)^{t-1}}$$

is the time-discounting weight of tertiary education relative to the resulting, additional wage benefit. As the Appendix shows, since education precedes wage benefits, $D_h(r)$ is increasing in r, as long as r is small enough to satisfy $r < r_h \equiv \sup\left\{r > 0 \,\middle|\, h\sum_{t=\theta_h+1}^{T}(1/1+r)^{t-1} > \sum_{t=\theta_m+1}^{T}(1/1+r)^{t-1}\right\}$ in which case tertiary education effectively raises "lifetime productivity". It is also readily shown that $p_e(r)/w_h(r)$ is increasing in r. Thus, we have the following lemma.

Lemma 1. *The threshold ability for tertiary education that separates middle- and high-skilled workers, i.e.,* $\bar{a}_h$ *defined in* (11'), *increases with the rental rate* r *for all the relevant levels of* r, *i.e.,* $0 < r < r_h$.

Proof. See Appendix A.1. □

The lemma shows that the mass of individuals who receive tertiary education decreases with r. On the one hand, an increase in the rental rate r raises the price for education p_e relative to the wage rate for high-skill labor w_h, as we can see from (17) that $p_e/w_h = r^{\frac{\alpha_e}{1-\alpha_m}}$. As (17b) indicates, an increase in the rental rate lowers the wage rate for high-skill labor, which in turn tends to lower the price for education. But the provision of educational services requires capital as well as high-skill labor, and hence the price for education relative to the wage rate for high-skill labor unambiguously increases with the rental rate while the price for education may or may not increase (see (17c)). On the other hand, the benefit of tertiary education in the form of an increase in future wages declines as the future is discounted more heavily. These two are the reasons why tertiary education becomes less attractive so the threshold ability increases if the rental rate (or equivalently the interest rate) increases.

The steady-state threshold ability for receiving secondary education is obtained from $\bar{I}_l(r, p_n) = \bar{I}_m(\bar{a}_l, r)$ as

$$\bar{a}_l(r, p_n) = D_e(r)\frac{\gamma p_e(r)}{w_h(r)} + D_l(r)\frac{w_l(r, p_n)}{w_h(r)}, \tag{10'}$$

where

$$D_e(r) \equiv \frac{\displaystyle\sum_{t=1}^{\theta_m}\left(\frac{1}{1+r}\right)^{t-1}}{\displaystyle\sum_{t=\theta_m+1}^{T}\left(\frac{1}{1+r}\right)^{t-1}} \quad \text{and} \quad D_l(r) \equiv \frac{\displaystyle\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^{t-1}}{\displaystyle\sum_{t=\theta_m+1}^{T}\left(\frac{1}{1+r}\right)^{t-1}} \tag{22}$$

are the time-discounting weight of secondary education relative to the wage benefit from secondary education and that of receiving low-skill wages relative to receiving high-skill wages, respectively. It directly follows from (17) that (i) w_l increases with p_n; (ii) p_e/w_h increases with r as shown above; and (iii) w_l/w_h increases with r under our assumption that $\alpha_n < \alpha_m$. In addition, similarly to the case of D_h, both D_e and D_l increase with r since receiving high-skill wages comes later in the lifetime, as the Appendix shows. Consequently, we have the following lemma.

Lemma 2. *The threshold ability for secondary education that separates low-skilled and middle-skilled workers, i.e.,$\bar{a}_l$ defined in* (10’)*, is increasing in the rental rate r under our assumption that $\alpha_n < \alpha_m$. It is also increasing in the price of the agricultural good p_n.*

Proof. See Appendix A.2. □

It is not surprising that an increase in the price of the agricultural good itself reduces the mass of individuals who receive secondary education through an increase in the relative wage rate for low-skill labor. As for the impact of the rental rate on the secondary-education threshold, we first note that an increase in the rental rate decreases the wage rate for high-skill labor relative to that for low-skill labor if the production of the manufactured good, which uses high-skill labor, is capital intensive compared with the production of the agricultural good, which uses low-skill labor. Second, as we have already shown, the price for education relative to the wage rate for high-skill labor increases with the rental rate. Together with the fact that an increase in the rental rate lowers the future benefits of increased wages, these imply that in the case where capital intensity is higher in manufacturing than in agriculture, education would be less attractive when the rental rate is high, so the threshold ability for receiving secondary education would increase.

Having derived individuals' educational choices in the steady state, we now calculate the aggregate supplies of production factors to obtain the steady-state version of the equilibrium conditions (20). The aggregate levels of low-skill labor, high-skill labor, and mass of students are given by

$$\bar{L}(r, p_n) = T \cdot F\big(\bar{a}_l(r, p_n)\big), \tag{14'}$$

$$\bar{H}(r, p_n) = (T - \theta_m) \int_{\bar{a}_l(r, p_n)}^{\bar{a}_h(r)} a \; dF(a) + (T - \theta_h) h \int_{\bar{a}_h(r)}^{\infty} a \; dF(a), \tag{15'}$$

$$\bar{S}(r, p_n) = \theta_m \left[F\big(\bar{a}_h(r)\big) - F\big(\bar{a}_l(r, p_n)\big)\right] + \theta_h \left[1 - F\big(\bar{a}_h(r)\big)\right]. \tag{16'}$$

It follows from Lemmas 1 and 2 that $\bar{L}$ increases while $\bar{H}$ and $\bar{S}$ decrease with r and p_n.

Using (6) and (8), we rewrite (13) to obtain the steady-state aggregate capital. The steady-state capital holding at an arbitrary time, say time T, of an individual whose ability is smaller than $\bar{a}_l(r, p_n)$ can be written as

$$\begin{aligned} \bar{K}_l(r, p_n) &\equiv \sum_{\tau=1}^{T-1} (1+r)^{T-1-\tau} \sum_{t=\tau}^{T-1} \left[\left(\frac{1}{1+r}\right)^{t-\tau} w_l(r, p_n) - \left(\frac{1}{1+r}\right)^{t-\tau} c_t(a, \tau) \right] && (23) \\ &= \sum_{\tau=1}^{T-1} (1+r)^{T-1-\tau} \sum_{t=\tau}^{T-1} \left[\left(\frac{1}{1+r}\right)^{t-\tau} w_l(r, p_n) - \left(\frac{1}{1+\rho}\right)^{t-\tau} \frac{\bar{I}_l(r, p_n)}{\Gamma_\tau(\tau)} \right], && (24) \end{aligned}$$

where we have used

$$c_t(a, \tau) = \left(\frac{1+r}{1+\rho}\right)^{t-\tau} \frac{1}{\Gamma_\tau(\tau)} \sum_{s=\tau}^{\tau+T-1} \left(\frac{1}{1+r}\right)^{s-\tau} w_l(r, p_n) = \left(\frac{1+r}{1+\rho}\right)^{t-\tau} \frac{\bar{I}_l(r, p_n)}{\Gamma_\tau(\tau)}. \tag{25}$$

We see from (25) that an individual's consumption expenditure increases over time when $r > \rho$, which holds true in equilibrium, and that the higher the rental rate r, the higher the growth rate of consumption. As a consequence, despite that an increase in r lowers the wage rate w_l, low-skilled workers save more in their early stage of life if r is high and hence the capital holding by all generations of low-skilled workers may well increase with the rental rate r. In addition, an increase in p_n raises w_l and hence the lifetime income $I_l(r, p_n)$. Thus, $\bar{K}_l(r, p_n)$ is increasing in p_n as we can see from (23).

Similarly, we can write the steady-state aggregate capital holdings by those who complete up to secondary education and those who complete tertiary education as functions of their ability and the rental rate:

$$
\begin{aligned}
\bar{K}_m(a, r) &\equiv \sum_{\tau=1}^{T-1} (1+r)^{T-1-\tau} \sum_{t=\tau}^{T-1} \left[\left(\frac{1}{1+r} \right)^{t-\tau} \bar{w}_m(a, \tau, t, r) - \left(\frac{1}{1+\rho} \right)^{t-\tau} \frac{\bar{I}_m(a, r)}{\Gamma_\tau(\tau)} \right], \\
\bar{K}_h(a, r) &\equiv \sum_{\tau=1}^{T-1} (1+r)^{T-1-\tau} \sum_{t=\tau}^{T-1} \left[\left(\frac{1}{1+r} \right)^{t-\tau} \bar{w}_h(a, \tau, t, r) - \left(\frac{1}{1+\rho} \right)^{t-\tau} \frac{\bar{I}_h(a, r)}{\Gamma_\tau(\tau)} \right],
\end{aligned}
$$

where $\bar{w}_i(a, \tau, t, r)$, for $i = m, h$ equals $-\gamma p_e$ for $t = \tau, \cdots, \tau + \theta_i - 1$, while it equals $w_h(r)a$ if $i = m$ and $w_h(r)ha$ if $i = h$ for $t = \tau + \theta_i, \cdots, \tau + T - 1$. The aggregate capital holding by middle-skilled and high-skilled workers may well increase with r for the same reason as for low-skilled workers.

The aggregate supply of capital is given by aggregating those capital holdings across all individuals with different abilities:

$$
\bar{K}(r, p_n) = \int_0^{\bar{a}_l(r, p_n)} \bar{K}_l(r, p_n) dF(a) + \int_{\bar{a}_l(r, p_n)}^{\bar{a}_h(r)} \bar{K}_m(a, r) dF(a) + \int_{\bar{a}_h(r)}^{\infty} \bar{K}_h(a, r) dF(a). \tag{13'}
$$

Now, the equilibrium conditions expressed by (20) can be written as a two-dimensional

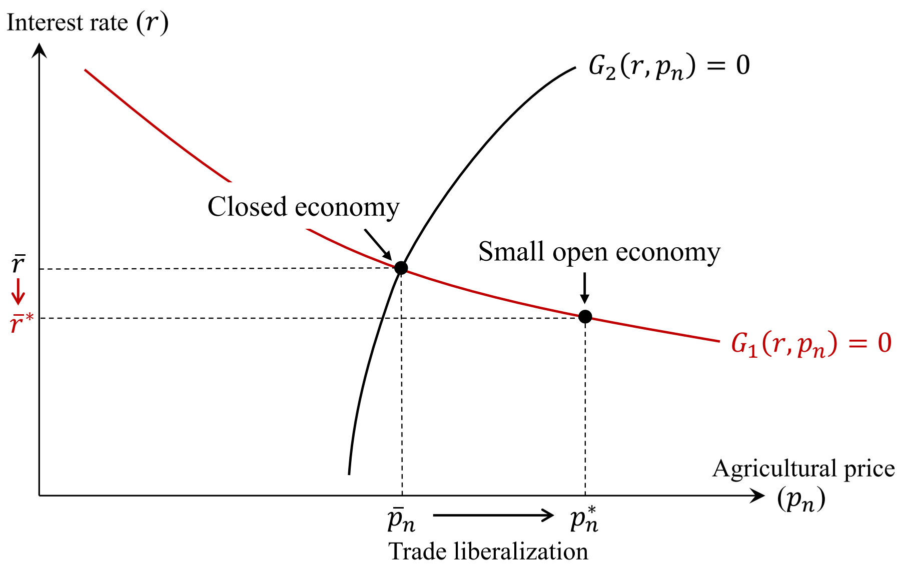

Figure 1: The Impacts of Trade on the Rental Rate

vector equation, $\mathbf{G}(r, p_n) = \mathbf{0}$, where

$$G_1(r,p_n) = \frac{\alpha_n}{1-\alpha_n}\omega_l(r,p_n)\bar{L}(r,p_n) + \frac{\alpha_m}{1-\alpha_m}\omega_h(r)\bar{H}(r,p_n) + \frac{\alpha_e-\alpha_m}{1-\alpha_m}\omega_h(r)^{1-\alpha_e}\gamma\bar{S}(r,p_n) - \bar{K}(r,p_n), \tag{20a'}$$

$$G_2(r,p_n) = \frac{\beta\bar{C}(r,p_n)}{p_n} - \frac{\omega_l(r,p_n)^{\alpha_n}\bar{L}(r,p_n)}{1-\alpha_n}. \tag{20b'}$$

The rental rate and price for the agricultural good in the autarkic steady-state equilibrium are illustrated as the intersection of the two curves in Figure 1. In our numerical simulations in the next section, we will focus on the case where the locus of $G_1(r, p_n) = 0$ is downward-sloping as depicted in the figure. The following lemma gives us the sufficient condition for it to be downward-sloping.

Lemma 3. *The steady-state excess demand for capital, $G_1(r, p_n)$, is decreasing in r if the conditions (i) and (iii) below hold, while it is decreasing in p_n if the conditions (ii) and* *(iii) hold.*

(i) $\int_0^{\bar{a}_l(r,p_n)} \frac{\partial \bar{K}_l}{\partial r}(r,p_n)\, dF(a) + \int_{\bar{a}_l(r,p_n)}^{\bar{a}_h(r)} \frac{\partial \bar{K}_m}{\partial r}(a,r)\, dF(a) + \int_{\bar{a}_h(r)}^{\infty} \frac{\partial \bar{K}_h}{\partial r}(a,r)\, dF(a) \geq 0.$

(ii) $\frac{\partial \bar{K}}{\partial p_n}(r,p_n) > \frac{\alpha_n}{1-\alpha_n} \frac{\partial \omega_l}{\partial p_n}(r,p_n) \bar{L}(r,p_n) \equiv \frac{\alpha_n}{(1-\alpha_n)^2} p_n^{\frac{\alpha_n}{1-\alpha_n}} r^{-\frac{1}{1-\alpha_n}} \bar{L}(r,p_n).$

(iii) The income shares of capital satisfy $\alpha_n \leq \alpha_m \leq \alpha_e$.

Proof. See Appendix A.3. □

We make a natural assumption, as we have argued above, that individuals' capital holdings, $\bar{K}_l(r,p_n)$, $\bar{K}_m(a,r)$, and $\bar{K}_h(a,r)$ increase with r due to their intertemporal consumption allocations so that Condition (i) is satisfied. The second inequality in Condition (iii) almost always fails to be met in reality, while the first inequality (which we assume to be true) is likely to hold in developing countries. Note that Condition (iii) is only a sufficient condition. Indeed, our numerical simulations, presented in the next section, suggest that condition (i) is satisfied for reasonable model parameters, and $G_1(r,p_n)$ is decreasing in r even when α_e is as small as zero so it is certainly smaller than α_n and α_m. Thus, we henceforth assume that $G_1(r,p_n)$ is decreasing in r.

As for the impact of p_n on $G_1(r,p_n)$, we find through the numerical simulation that if α_n is small enough, Condition (ii) is satisfied and $G_1(r,p_n)$ is decreasing in p_n even when the second inequality in Condition (iii) is widely violated. But, if α_n is large enough, Condition (ii) is violated, and $G_1(r,p_n)$ becomes increasing in p_n for a reasonable set of parameter values. When p_n increases due to trade liberalization for example, w_l increases in response, and it increases significantly if α_n is large, as shown in (17a). Capital substitutes for low-skill labor in agriculture, and the resulting increase in demand for capital can outweigh an increase in capital supply if the capital income share in agricultural production, α_n, is large. The locus of the capital-market clearing conditions, $G_1(r,p_n) = 0$, is downward-sloping as depicted in Figure 1 if α_n is so small that Condition (ii) and (iii) are satisfied, while it can be upward-sloping otherwise.

2.3 The Impact of Trade Liberalization

Since we are interested in the impact of trade liberalization on a developing country, we assume that the country is small and exports agricultural goods while importing manufactured goods. This section considers the long-run impact of an increase in the price for agricultural goods as a result of trade liberalization, i.e., we compare the steady-state equilibrium in free trade with that in autarky. The impacts are different between the case where the capital income share in the agricultural sector is so small that the locus of $G_1(r, p_n) = 0$ is downward-sloping and the case where it is large enough that the locus is upward-sloping. Our numerical simulations in the next section suggest that the first case is more plausible but the second case is not implausible even in developing countries that are likely to use labor-intensive techniques in agriculture. We will examine the impact of trade liberalization including the transition phase to a new steady state in the next section.

2.3.1 Case 1: α_n is so small that the locus of $G_1(r, p_n) = 0$ is downward-sloping

In this case, G_1 is decreasing in both r and p_n so that the steady-state rental rate r drops as p_n increases from the autarkic level to the world price p_n^* as a result of trade liberalization, as depicted in Figure 1. We will examine how this change affects individuals' educational choices and income distribution within the country.

It directly follows from Lemma 1 that a decrease in the rental rate caused by trade liberalization lowers the ability threshold of tertiary education, increasing the mass of high-skilled workers. Lemma 2 implies that the ability threshold of secondary education increases if a disincentive to receive secondary education as a consequence of an increase in w_l caused directly by an increase in p_n outweighs an increase in the attractiveness of receiving secondary education as a result of a decrease in r. Thus, we have the following proposition.

Proposition 1. *Consider the case where the capital income share in the agricultural* *sector is small such that the rental rate decreases as a result of trade liberalization. Then, the mass of individuals who receive tertiary education is greater in free trade than in autarky. The mass of individuals who receive secondary education or higher declines if the direct, positive impact of an increase in the price of agricultural goods on the wage rate for low-skill labor outweighs the effect of a decrease in the rental rate that enhances the attractiveness of higher education.*

Proof. The proposition obtains directly from Lemmas 1 and 2. □

The direct impact of trade liberalization, which benefits low-skilled workers through an increase in the price for agricultural goods, likely outweighs the indirect effect through a decline in the rental rate. Thus, trade liberalization is likely to lead to education polarization caused by lifetime income polarization.

Now, let us examine how opening to trade affects steady-state income inequality between and within the three groups of workers. We first look at the effect on the wage gaps and then examine the lifetime income inequality.

Measuring the wage gap between different worker groups by the ratio of the wage rates, we obtain from (17) the wage gap between the low-skilled workers and the middle-skilled workers with an ability of a as

$$\frac{w_h a}{w_l} = r^{\frac{\alpha_n - \alpha_m}{(1-\alpha_m)(1-\alpha_n)}} p_n^{-\frac{1}{1-\alpha_n}} a. \tag{27}$$

With the good prices being fixed, a decrease in r will increase the wage gap under our assumption that $\alpha_n < \alpha_m$, while an increase in p_n will reduce it.[4] Thus, the effect of trade liberalization on the wage gap between low-skilled and middle-skilled workers is ambiguous; trade liberalization reduces the wage gap if the direct impact of an increase in p_n outweighs

[4]In any industry, the positive impact of a decrease in r on the wage rate is greater when the capital income share is large, as argued in the discussion that follows Lemma 2.

the opposite effect of a decrease in r.

The wage gap between the middle-skilled workers with an ability of a and the high-skilled workers with an ability of a' can be measured by $w_h ha'/w_h a = ha'/a$, and it would not be affected by trade liberalization. The wage gap between low-skilled workers and high-skilled workers will become smaller if and only if the wage gap between low-skilled and middle-skilled workers shrinks as a consequence.

What about the effect on lifetime income inequality? Let us first look at within-group income inequality. First, note the lifetime income of low-skilled workers does not depend on their ability, so there is no income inequality among low-skilled workers before and after trade liberalization. To see a change in income inequality among middle-skilled workers, we rewrite $\bar{I}_m(a, r)$ using (21) and (22) as

$$\bar{I}_m(a, r) = w_h(r) \sum_{t=\theta_m+1}^{T} \left(\frac{1}{1+r}\right)^{t-1} \left[a - D_e(r)\frac{\gamma p_e(r)}{w_h(r)}\right]. \tag{28}$$

Then, the lifetime income gap between the middle-skilled workers with abilities a and $a'(> a)$ can be written as

$$\frac{\bar{I}_m(a', r)}{\bar{I}_m(a, r)} = \frac{a' - D_e(r)\frac{\gamma p_e(r)}{w_h(r)}}{a - D_e(r)\frac{\gamma p_e(r)}{w_h(r)}}. \tag{29}$$

It follows from $D_e(r)$ and $p_e(r)/w_h(r) = r^{\frac{\alpha_e}{1-\alpha_m}}$ are both increasing in r that trade liberalization, which leads to a decline in r, lowers the cost of education relative to future earnings. Since this relative cost is greater for lower-ability workers (who receive lower rewards for education) than higher-ability workers among the middle-skilled workers, a decline in the cost of education relative to future earnings disproportionately benefits workers with low ability. That is why trade liberalization lowers income inequality among middle-skilled workers. It is easy to see that the same argument goes through for high-skilled workers; trade liberalization reduces the lifetime income inequality also among high-skilled workers.

Turning to the lifetime income inequality between different groups of workers, we first observe that the inequality between the low-skilled workers and the middle-skilled workers with an ability of a can be written as

$$\frac{\bar{I}_m(a,r)}{\bar{I}_l(r,p_n)} = \frac{\bar{I}_m(a_l,r)}{\bar{I}_l(r,p_n)} \frac{\bar{I}_m(a,r)}{\bar{I}_m(a_l,r)}. \tag{30}$$

Consider the case where the effect of an increase in p_n outweighs that of a decline in r, so that trade liberalization decreases the first term on the right-hand side with a_l fixed at the closed-economy level. The second term also decreases in this case, so we find that the lifetime income inequality between low-skilled and middle-skilled workers declines.

Next, we write the lifetime income gap between middle-skilled workers with ability a and high-skilled workers with ability a' as

$$\frac{\bar{I}_h(a',r)}{\bar{I}_m(a,r)} = \frac{\bar{I}_m(a_h,r)}{\bar{I}_m(a,r)} \frac{\bar{I}_h(a_h,r)}{\bar{I}_m(a_h,r)} \frac{\bar{I}_h(a',r)}{\bar{I}_h(a_h,r)}. \tag{31}$$

With a_h being fixed at the closed-economy level, the second-term increases (Proposition 1) while the other within-group inequalities decrease. Therefore, the total effect of trade liberalization on the lifetime income inequality between middle-skilled and high-skilled workers is ambiguous.

We record these findings as a proposition.

Proposition 2. *Consider the case where the capital income share in the agricultural sector is so small that the rental rate decreases as a result of trade liberalization. Then, trade liberalization reduces the steady-state income inequality, in terms of both the wage rate and the lifetime income, between the low-skilled and middle-skilled workers if the effect of an increase in the price of agricultural goods on the economy outweighs that of a decline in the rental rate. The wage gap between middle-skilled and high-skilled workers does not change as a result of trade liberalization. The effect on the lifetime income inequality between* *middle-skilled and high-skilled workers is ambiguous. Trade liberalization, however, reduces the lifetime income gap within the income groups.*

2.3.2 Case 2: α_n is so large that the locus of $G_1(r, p_n) = 0$ is upward-sloping

In this case, trade liberalization entails a rise in the rental rate r. Lemma 1 shows that the threshold of tertiary education increases as a result of trade liberalization. Lemma 2 implies that under our assumption that $\alpha_n < \alpha_m$, an increase in p_n and an increase in r both contribute to a rise in the threshold for secondary education. Thus, we have the following counterpart of Proposition 1.

Proposition 3. *Consider the case where the capital income share in the agricultural sector is so large that the rental rate increases as a result of trade liberalization. Then, the mass of individuals who receive tertiary education is smaller in free trade than in autarky. Trade liberalization raises the threshold of secondary education if the capital income share is greater in manufacturing than in agriculture, in which case the mass of individuals who receive only up to primary education increases.*

Proof. The proposition obtains directly from Lemmas 1 and 2. □

An increase in r generally discourages individuals from receiving higher education. Therefore, trade liberalization likely lowers the education level of the country if the steady-state rental rate becomes higher than in autarky.

As for the effect on the wage gap between the low-skilled workers and the middle-skilled workers, we find immediately from (27) that trade liberalization unambiguously reduces the wage gap under our assumption that $\alpha_n < \alpha_m$. It does not affect the wage gap between middle-skilled workers and high-skilled workers as in the previous case.

Unlike the wage gap, the effect on the lifetime income gap between low-skilled and middle-skilled workers is ambiguous because trade liberalization lowers the first term on the right-hand side of (30) while it raises the second term. Similarly, the impact on the lifetime income inequality between middle-skilled and high-skilled workers is ambiguous since the second term on the right-hand side of (31) drops while the first and third terms increase.

We record our findings in the following proposition.

Proposition 4. *Consider the case where the capital income share in the agricultural sector is so large that the rental rate increases as a result of trade liberalization. If the capital income share is greater in manufacturing than in agriculture, trade liberalization reduces the steady-state wage gap between low-skilled and middle-skilled workers. Trade liberalization widens the lifetime income inequality both among middle-skilled workers and among high-skilled workers. However, the impact on the lifetime income inequality between the groups is ambiguous.*

2.3.3 Comparison between the two cases

The capital income share in agriculture production is likely to increase as the economy develops because the advanced economy usually has better access to capital-intensive techniques and economic development tends to increase the wage rate, inducing producers to adopt more capital-intensive production techniques. Our analysis indicates that the effect of trade liberalization on income distribution and individuals' choice of education critically depends on the size of the capital income share in agriculture production. In relatively-developed countries where the capital income share is large in agriculture, trade liberalization would likely raise the rental rate and hence discourage individuals from getting more education. This observation generally accords with our intuition that trade liberalization that facilitates the low-skill-intensive agricultural sector discourages education. We find, however, that in less well-off developing countries where the capital income share in agriculture is small, trade would lower the rental rate, which encourages individuals to get more education. This stark contrast is important when developing countries contemplate opening to trade.

Table 1: Parameter Values

	Value	Reference
T	50	Assumption
T_L	70	Assumption
α_n	0.3840	Bulgaria in 2019, wiiw Growth and Productivity Data (Release Jan 2022)
α_m	0.3947	Bulgaria in 2019, wiiw Growth and Productivity Data (Release Jan 2022)
α_e	0.1959	Bulgaria in 2019, wiiw Growth and Productivity Data (Release Jan 2022)
δ	0.0532	Bulgaria in 2019, Penn World Table version 10.0 from Feenstra (2015)
β	0.5	Assumption
θ_m	6	Assumption (length of secondary education)
θ_h	10	Assumption (length of secondary and tertiary education)
h	1.278	Sub-Saharan Africa, Psacharopoulos (1994)
ρ	0.0322	Calibration
γ	78.7500	Calibration
μ	2.3041	Calibration
σ	1.4555	Calibration

3 The Effect of Trade Liberalization: A Numerical Analysis of Transition Dynamics

Comparing the two steady states, one in the closed economy and the other in free trade, does not give us a complete picture of the impact of trade liberalization on income distributions within the developing countries, since individuals including those in the future generations adjust their educational decisions accordingly so the impact will be gradual, and the adjustment can last for decades. This section numerically analyzes the transition of the economy that unexpectedly opens to trade in period 0.

3.1 Calibration

Table 1 lists all the parameter values assigned in our numerical analysis. The length of the working period T is 50, representing the number of years after primary education and before retirement. In our simulation, we introduce the after-retirement period of 20 years (in which individuals only consume goods) so that the total lifetime is $T_L = 70$ years after secondary education, in order to make our calibration as realistic as possible. In addition, we also consider capital depreciation in our calibration. Due to the limited data availability for developing countries, we use data for Bulgaria (instead of a Sub-Saharan country for example) for capital depreciation rate, denoted by δ, and capital income shares.[5] The agriculture-consumption expenditure share β is assumed to be 0.5. The length of secondary education, θ_m, and that of secondary and tertiary education, θ_h, are set at 6 years and 10 years, respectively. We use Sub-Saharan African private return to higher education of $h = 1.278$ reported by Psacharopoulos (1994) for the productivity premium of high-skilled workers relative to the middle-skilled.

The other parameters are derived from calibration. All those targets are shown in Table 2. We use the Sub-Saharan African 5-year average real interest rate in 2017-2021 for the interest (rental) rate r. Due to the limited data availability, we use the Latvian data as the education spending share in GDP as Latvia's share was the lowest in OECD countries in 1995 (OECD iLibrary). The shares of individuals that receive secondary and tertiary education come from non-advanced, developing country data in Barro and Lee (2013).[6] An individual's ability is assumed to be distributed as log-normal such that

$$f(a) = \frac{1}{\sqrt{2\pi}\sigma a} \exp\left(-\frac{(\log a - \mu)^2}{2\sigma^2}\right).$$

Table 2: Calibration Targets

	Data	Model	Reference
Rental rate r	0.0485	0.0431	Sub-Saharan Africa in 2017-2021, World Bank (2022)
Education spending share	0.0190	0.0185	Latvia in 1995, OECD (2022)
Share of short-term educ.	0.1710	0.1735	Developing counties in 1990, Barro and Lee (2013)
Share of long-term educ.	0.0290	0.0286	Developing counties in 1990, Barro and Lee (2013)

[5]Bulgaria's real GDP per capita was the lowest in the European Union in 2019 (Penn World Table, version 10.0).

[6]From Table 3 (Developing Region in 1990) in Barro and Lee (2013), the share of those who received secondary education is calculated to be the share of individuals who completed secondary education (14.4%) plus those who dropped out of tertiary education (5.6% – 2.9%), while the share of individuals who received tertiary education is the share of individuals who completed tertiary education (2.9%).

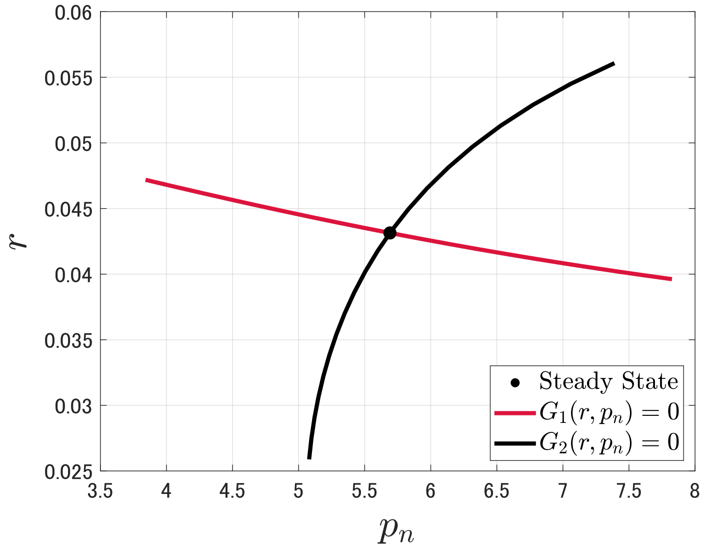

Figure 2: The Steady State in Closed Economy

We calibrate parameters $(\rho, \gamma, \mu, \sigma)$, as reported in Table 1, so as to be consistent with the data in Table 2, based on the closed-economy steady state. Table 2 summarizes the results.

Figure 2 shows the downward-sloping capital-market clearing schedule, i.e., the locus of $G_1(r, p_n) = 0$, and the upward-sloping agricultural-good-market clearing schedule, i.e., the locus of $G_2(r, p_n) = 0$. Note that the economy we analyze here is in Case 1 in the last section. The intersection of these curves depicts the steady-state equilibrium of the closed economy.

3.2 Effect of Trade Liberalization

We simulate the impact of trade liberalization that induces a 20% hike in the price of agricultural goods from an autarkic price of $\bar{p}_n$ to a world price of $p_n^* = 1.2\bar{p}_n$ on the transition paths of the capital stock, rental rate, wage rates, and individuals' educational choices and their consequences on the composition of low-skilled, middle-skilled, and high-skilled workers. We also derive changes in individuals' utilities and social welfare.

Figure 3 illustrates the transition paths of the capital, rental rate, and wage rates, while Table 3 shows the percentage changes in these variables from the autarkic steady state at

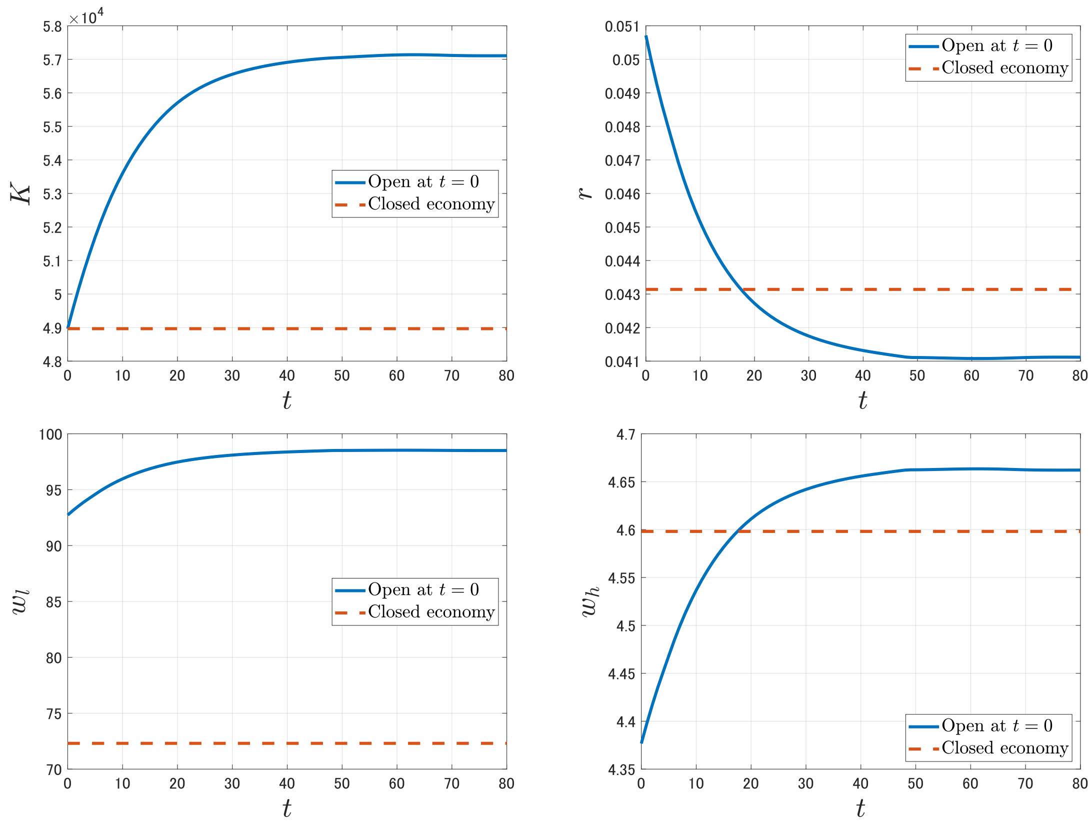

Figure 3: Capital Accumulation, Rental Rate, and Wages

$t = 0, 10, 20$ and the new free-trade steady state, i.e., $t = \infty$.

When the price of agricultural goods rises at time 0, the wage rate for low-skill labor and the rental rate, which are the prices of the factors used in the production of agricultural goods, jump up. As a consequence of a rise in the rental rate, capital gradually accumulates toward the new steady-state level. As capital accumulates, however, the rental rate goes down and will become lower than the autarkic level at $t = 18$ in this scenario. The wage rate for high-skill labor jumps down at $t = 0$ (due to an increase in r) and will gradually increase as capital, which is combined with high-skill labor to produce manufactured goods and educational services, accumulates and will surpass the autarkic level also at $t = 18$. The wage rate for low-skill labor also gradually increases as capital accumulates.

Table 3: Changes in Variables from the Closed-economy Steady State (% change)

	$t = 0$	$t = 10$	$t = 20$	$t = \infty$
r	17.56	4.65	−0.97	−4.71
w_l	28.25	32.73	34.80	36.24
w_h	−4.81	−1.34	0.28	1.40
K	0.00	9.47	13.76	16.65
L	0.14	1.00	1.82	4.03
H	−0.02	−0.95	−2.04	−4.32

3.2.1 Educational Choice

Figure 4 and Table 4 show the effects of trade liberalization on educational choice, in particular on the cutoff levels of ability and the changes in the low-skill and high-skill labor.

As discussed in the last section, the steady-state masses of individuals who receive only primary education and those who received tertiary education increase while the mass of those who receive up to secondary education decreases as a result of opening to trade, as shown by changes in generation $\tau = \infty$ in Table 4. The mass of individuals who only receive primary education jumps up by 4.45% in the generation immediately after trade liberalization, due to the sharp increase in the cutoff a_l that follows the opening to trade. The mass of individuals who receive tertiary education gradually increases over time, as depicted in the northwest panel of Figure 4 shows.[7] A gradual decline in the rental rate, which follows a hike due to trade liberalization, explains why the cutoffs, a_h and a_l, decrease gradually. A declining rental rate

Table 4: Changes in Educational Choice from the Closed-economy Steady State (% change)

	$\tau = -6$	$\tau = 0$	$\tau = \infty$
Primary education	0.00	4.45	4.03
Secondary education	−0.84	−23.13	−23.46
Tertiary education	5.09	16.30	29.82

[7]Since the masses of individuals that receive secondary or tertiary education are small compared to those who only received primary education, the percentage changes of the former are significantly greater than the latter, as indicated in Table 4.

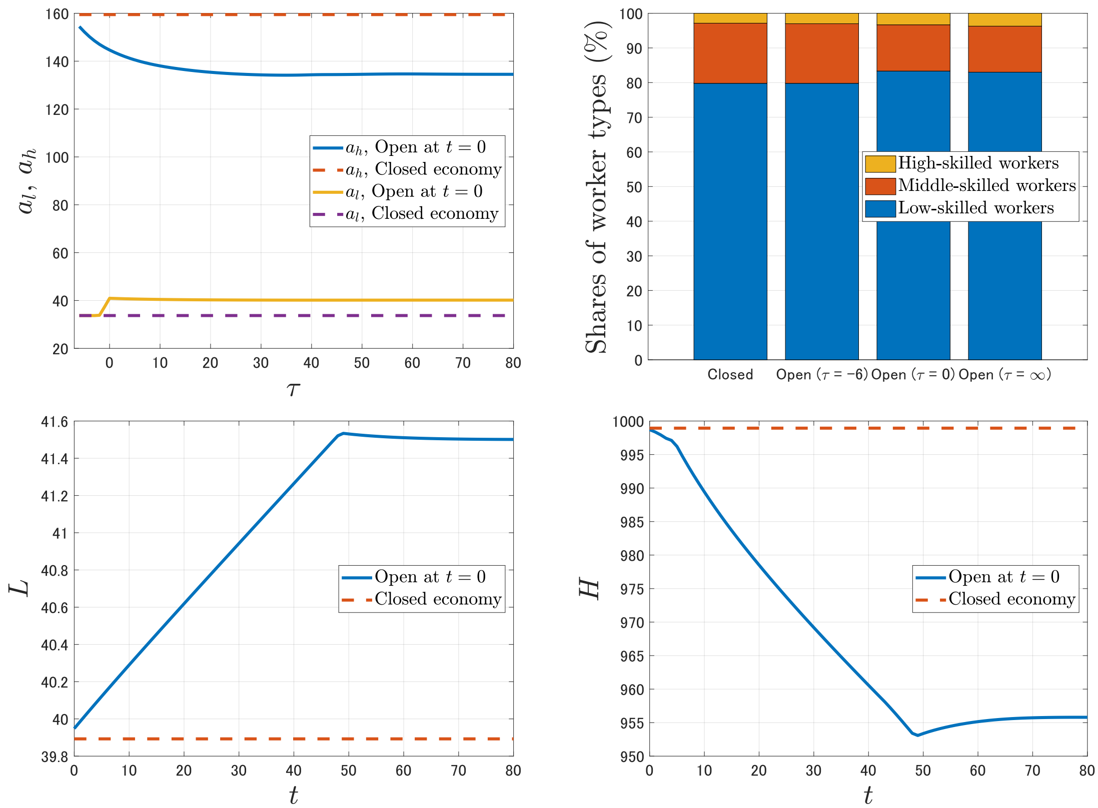

Figure 4: Factor Supplies and Cutoff Levels of Ability

is associated with a gradual increase in the wage rate for high-skill labor. Anticipating future increases in the wage rate, more and more individuals choose to receive tertiary education. This gradualism shows a stark difference from the immediate increase in the cutoff a_l.[8] Table 4 also indicates that some of those who have just finished secondary education (in generation $\tau = -6$) choose to proceed to tertiary education instead of working right away. In the end, the population that receives tertiary education increases by as much as 29.82%.

The northeast panel of Figure 4 illustrates the population shares of low-skilled, middle-skilled, and high-skilled workers, respectively, which reflect the evolution of individuals' educational choices depicted in the northwest panel. We see clearly that trade liberalization entails education polarization, i.e., middle-skilled workers are squeezed out by the expansions

[8]The cutoff a_l slightly decreases as well in the transition path because the decline in the rental rate raises the wage rate for the high-skill labor.

of low-skilled and high-skilled workers. The expansion of the agricultural sector as a result of trade liberalization immediately and significantly increases the population that does not proceed to secondary education. The indirect effect through a decline in the interest rate on educational choice is relatively small and only appears gradually.

As a consequence, the aggregate low-skill labor, denoted by L, keeps increasing up to $t = 49$ where all low-skilled workers at the time of trade liberalization have retired, as illustrated in the southwest panel. Then L starts declining (slightly) reflecting a gradual decline in the population that does not proceed to secondary education after trade liberalization. The evolution of the aggregate high-skill labor, denoted by H, is just opposite to that of L, as indicated in the southeast panel.

3.2.2 Welfare

In general, international trade benefits a country as a whole but hurts individuals who possess production factors that are intensively used in the import goods. In addition, the impacts can be different across different generations even within the same skill groups.

Figure 5 shows individual lifetime utilities, defined by (2), for a low-skilled, middle-skilled ($a_1 \equiv (a_l + a_h)/2 = 96.58$), and a high-skilled ($a_2 \equiv a_h + (a_h - a_l)/2 = 222.32$) workers in generations from $\tau = -69$ to $\tau = 60$. Opening to trade increases the lifetime utility for low-skilled workers for almost all generations. As panel (a) indicates, the lifetime utility is lowest for generation $\tau = -50$ who has just retired at the time of trade liberalization. They hold the greatest capital goods as savings than those in other generations and are severely affected by a decrease in the relative price for capital goods, or equivalently manufactured goods. After generation $\tau = -50$, the later the generation, the more benefits they get from opening to trade, due to the longer tenure and a declining interest rate.

Panels (b) and (c) of Figure 5 show the lifetime utilities of middle- and high-skilled workers, respectively. They are worse off by trade liberalization. Generations that are in

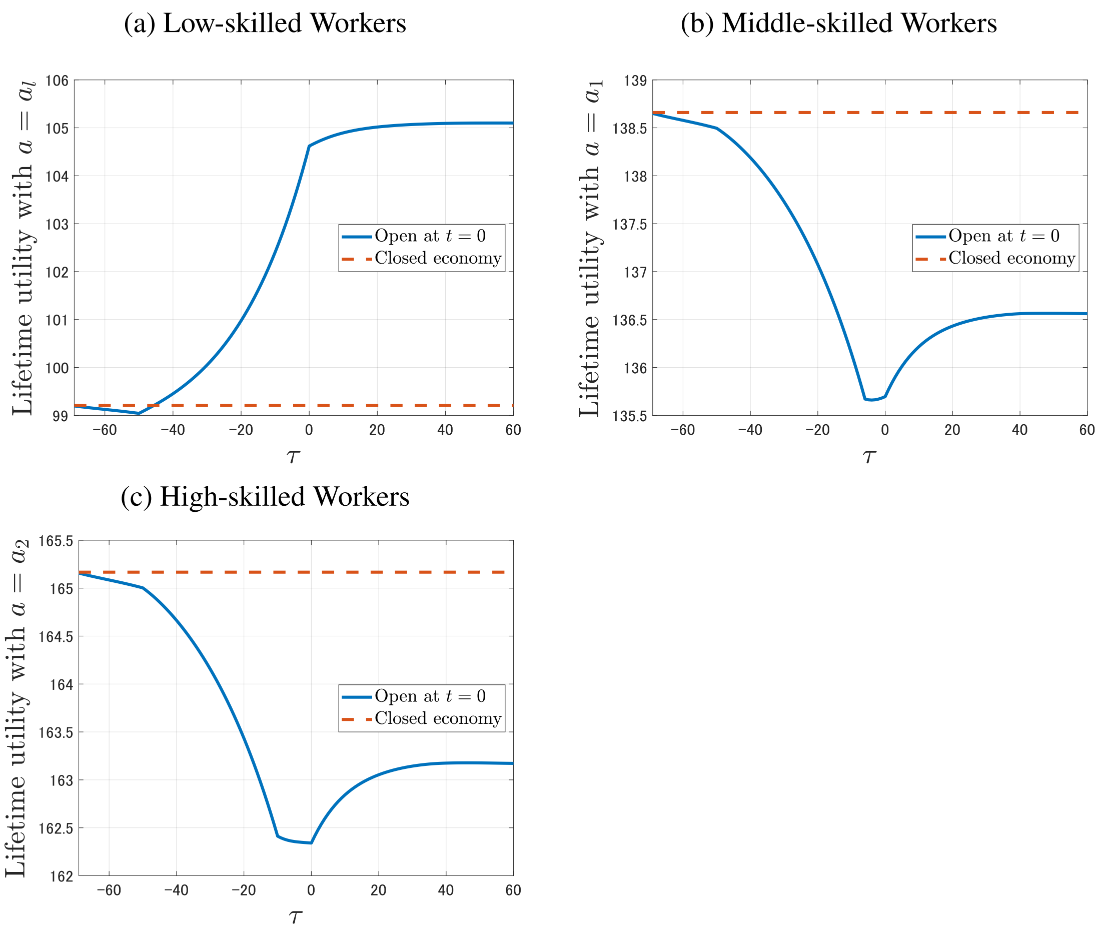

Figure 5: The Impact of Trade Liberalization on Individuals' Lifetime Utilities

school at the time of trade liberalization, generations from $\tau = -6$ to $\tau = 0$ for middle-skilled workers and those from $\tau = -10$ to $\tau = 0$ for high-skilled workers, are most severely hurt. Generations before them had enjoyed higher wages before $t = 0$, while those after them enjoy lower costs of education and a declining interest rate.

What about the impact on social welfare? We measure social welfare by the average instantaneous utility of all the individuals in each period. As Figure 6 illustrates, social welfare drops slightly when the country opens to trade. This is because individuals reduce consumption to increase savings in response to a hike in the interest rate. Social welfare gradually increases and quickly surpasses the level of autarky.

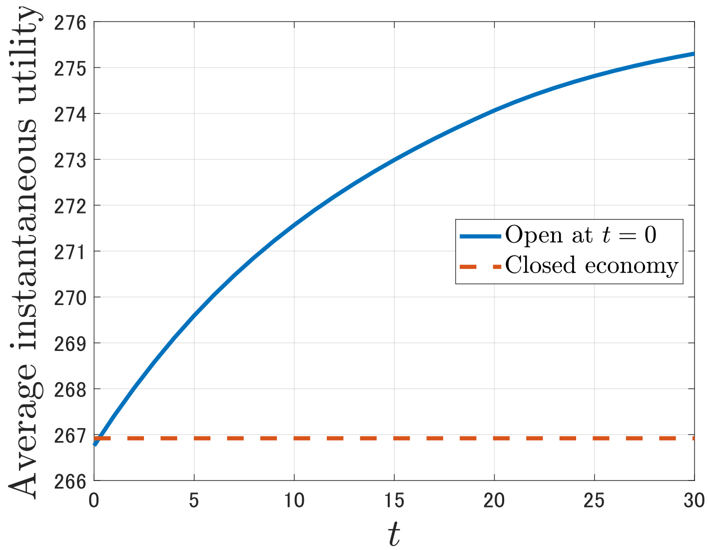

Figure 6: The Impact of Trade Liberalization on the Social Welfare

3.2.3 Income Inequality

In the last section, we compared within- and between-group income inequalities between the two steady states, before and after trade liberalization, and found that the effect of trade liberalization on income inequality is generally ambiguous, depending on the relative strength of economic factors in many cases. Here, we show those effects in our simulated economy and investigate how these effects vary across different generations.

First, we examine the impact of trade liberalization on the lifetime income gaps between a low-skilled worker and a middle-skilled worker with ability $a_1 = 96.58$. Figure 7 illustrates how trade liberalization changes income inequality for individuals in various generations through the between- and within-group effects, and the total effect as the sum of them. The figure shows that trade liberalization reduces income inequality between the low-skilled and middle-skilled workers, measured by $I_m(a_1, \tau)/I_l(\tau)$, for all generations but the retirees, with the greatest impact on generation $\tau = 0$. The impact can be decomposed into the between-group income inequality, measured by $I_m(a_l, \tau)/I_l(\tau)$, and the within-group income inequality, measured by $I_m(a_1, \tau)/I_m(a_l, \tau)$. It is readily seen that income inequality shrinks through the between-income effect for all generations but retirees, due to an increase in the wage rate for low-skill labor as a result of an increase in the price for agricultural goods, and this effect outweighs the within-group effect. A change in the between-group effect is greatest in magnitude for generation $\tau = 0$, in which the low-skilled workers benefit from the very beginning of their career from a rise in the price for agricultural goods. Low-skilled workers in earlier generations get such benefits only from their mid-career, while those in later generations obtain smaller wage benefits from trade liberalization relative to middle-skilled workers (since we assume $\alpha_m > \alpha_n$) because the interest rate declines over time.

On the contrary, income inequality rises in the generations born earlier than the time of trade liberalization, while it shrinks in later generations through the within-group effect. The within-group income inequality (among middle-skilled workers) is greatest in generation $\tau = -6$, which faces a large decline in the wage rate at the very beginning of their career. The discounted sum of wages is the lowest among all the generations, and hence the education burden relative to its rewards is greatest. This education burden hurts workers with lower skills (and thus lower wages) more, so income inequality rises most significantly for generation $\tau = -6$. The within-group effect, which is caused indirectly by trade liberalization through an increase in the rental rate, is much smaller and hence outweighed by the directly caused between-group effect.

Figure 8 indicates that the between- and within-group effects are similar in magnitude for the impact on the lifetime income inequality between a middle-skilled worker with ability $a_1 = 96.58$ and a high-skilled worker with ability $a_2 = 222.32$, as both effects arise from indirectly induced changes in the rental rate. In this exercise, trade liberalization decreases income inequality between the middle-skilled workers and the high-skilled workers for all generations older than $\tau = -8$, while it increases for all later generations. Income inequality shrinks most for generation $\tau = -10$, and it increases most for generation $\tau = -5$.

Although similar in magnitude, the overall impact on income inequality is driven mostly

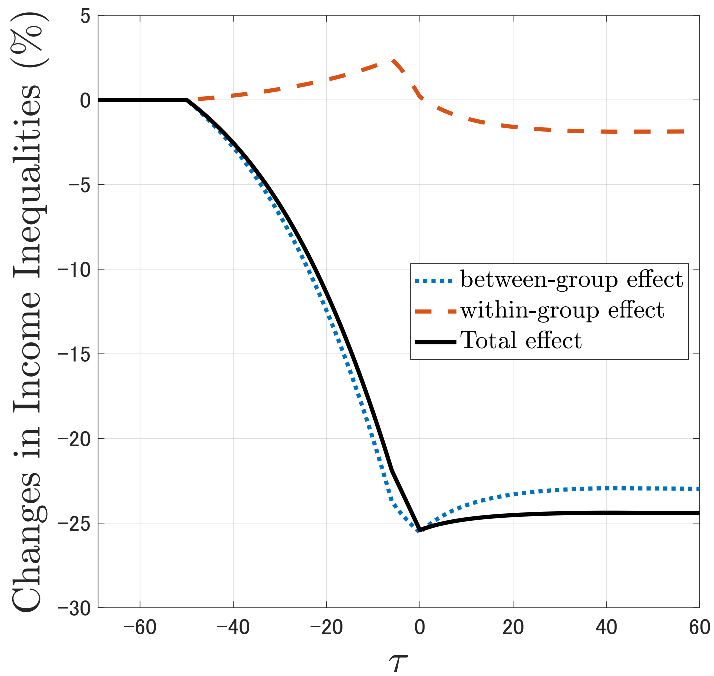

Figure 7: The Impact on Income Inequality between low-skilled and middle-skilled workers

by the between-group effect also in this case. For generation $\tau = -10$ or earlier, they have already started working at the time of trade liberalization. So, the trade-induced hike in the interest rate reduces the discounted sum of their income from then on and hence reduces the lifetime income gap between middle-skilled and high-skilled workers. This between-group effect is greatest for generation $\tau = -10$, in which those who receive tertiary education have just graduated from college at the time of trade liberalization. In generations from $\tau = -10$ to $\tau = -6$, middle-skilled workers are working, while prospective high-skilled workers are still at school when trade is liberalized. The hike in the rental (or interest) rate reduces the high-skill wage rate more than it lowers the educational costs. This effect contributes to the expansion of the lifetime income gap between the two groups. The income gap increases through the between-group effect in later generations as the interest rate declines and becomes lower than the pre-liberalization level. The within-group effect, which combines the within-group effect for middle-skilled workers and that for high-skilled workers, is similar to the case between low-skilled and middle-skilled workers and outweighed by the between-group effect also in

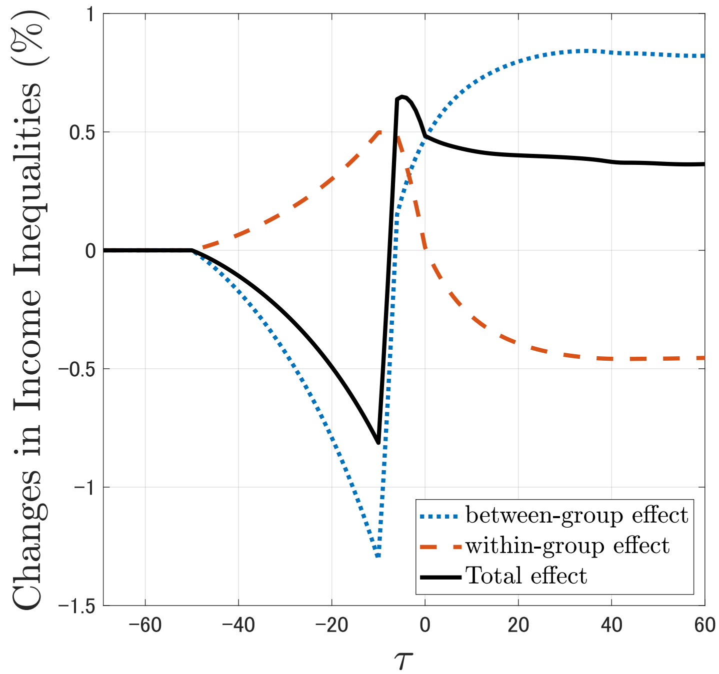

Figure 8: The Impact on Income Inequality between middle-skilled and high-skilled workers

this case.

4 Conclusion

International trade is beneficial to each engaging country as a whole, allowing it to specialize in its comparative advantage sectors and enhance its real income as a consequence. However, the story is not that simple for developing countries, which usually have a comparative advantage in agricultural and low-tech manufacturing sectors, since trade liberalization tends to take resources away from capital- and knowledge-intensive manufacturing industries, which serve as an engine of economic growth (e.g., Matsuyama, 1992).

In this paper, we have investigated the effect of trade liberalization on income distribution and educational choice in developing countries. In particular, we explicitly incorporate individuals' choice of receiving secondary and tertiary education in addition to primary education so that we can examine the short-run and long-run impacts on the supply side of the labor market as well as the demand side in more detail than in a typical model with low-skilled and high-skilled workers. Then, we have shown that in developing countries with a low capital intensity in the agricultural sector, trade liberalization leads to education polarization such that the proportion of high-skilled workers, who received tertiary education, increases in the long run as well as low-skilled workers. This result is consistent with Blanchard and Olney (2017) empirical finding that an increase in agricultural exports does not reduce the population that receives tertiary education in developing countries. Even though trade liberalization increases the population that receives only up to primary education, the observation that it can also increase the population that receives tertiary education is noteworthy because those highly educated individuals can contribute to economic growth in the long run.

Whether or not trade liberalization is good in the long run for developing countries critically depends on its effect on individuals' educational choices. The current paper focuses on the impact on income distribution and educational choice, abstracting away the growth aspects. We leave the investigation of the effect of trade liberalization in the context of economic growth for future research.

A Appendix

A.1 Proof of Lemma 1

We show that the threshold ability for tertiary education, $\bar{a}_h(r) = D_h(r)\left[\gamma p_e(r)/w_h(r)\right]$, increases with r. To this end, we first note that $p_e(r)/w_h(r) = r^{\alpha_e/(1-\alpha_m)}$, derived from (17b) and (17c), increases with r. We readily see that $D_h(r)$ is also increasing in r as it can be rewritten as

$$
\begin{aligned}
D_h(r) &\equiv \frac{\displaystyle\sum_{t=\theta_m+1}^{\theta_h}\left(\frac{1}{1+r}\right)^{t-1}}{\displaystyle h\sum_{t=\theta_h+1}^{T}\left(\frac{1}{1+r}\right)^{t-1} - \sum_{t=\theta_m+1}^{T}\left(\frac{1}{1+r}\right)^{t-1}} \\
&= \frac{\displaystyle\sum_{t=\theta_m+1}^{\theta_h}(1+r)^{\theta_h-t}}{\displaystyle(h-1)\sum_{t=\theta_h+1}^{T}\left(\frac{1}{1+r}\right)^{t-\theta_h} - \sum_{t=\theta_m+1}^{\theta_h}(1+r)^{\theta_h-t}}.
\end{aligned}
\tag{A.1}
$$

Thus, we have shown that $\bar{a}_h(r)$ increases with r.

It is also easy to see that r cannot increase indefinitely without completely eliminating an incentive to receive tertiary education. The denominator of (A.1) is decreasing in r and becomes negative when $r > r_h$. Indeed, we see that the productivity increment as a consequence of tertiary education is positive, i.e.,

$$\sum_{t=\theta_h+1}^{T} h\left(\frac{1}{1+r}\right)^{t-1} - \sum_{t=\theta_m+1}^{T}\left(\frac{1}{1+r}\right)^{t-1} = \left(\frac{1}{1+r}\right)^{\theta_h-1}\left[(h-1)\sum_{t=\theta_h+1}^{T}\left(\frac{1}{1+r}\right)^{t-\theta_h} - \sum_{t=\theta_m+1}^{\theta_h}(1+r)^{\theta_h-t}\right] > 0,$$

if and only if the denominator of (A.1) is positive. This implies that no one would receive tertiary education if $r > r_h$.

A.2 Proof of Lemma 2

We show that

$$\bar{a}_l(r, p_n) = D_e(r)\frac{\gamma p_e(r)}{w_h(r)} + D_l(r)\frac{w_l(r, p_n)}{w_h(r)}$$

is increasing in p_n, and under our assumption that $\alpha_n < \alpha_m$ in r as well.

First, we see that $\bar{a}_l$ increases in p_n because $w_l(r, p_n)/w_h(r)$ increases with p_n, as observed from (17a) and (17b).

As for the effect of r, we first note that $D_e(r)$ and $D_l(r)$ are increasing in r, as readily seen from

$$D_e(r) \equiv \frac{\sum_{t=1}^{\theta_m} \frac{\gamma}{(1+r)^{t-1}}}{\sum_{t=\theta_m+1}^{T} \frac{h}{(1+r)^{t-1}}} = \frac{\gamma \sum_{t=1}^{\theta_m} (1+r)^{\theta_m - t}}{h \sum_{t=\theta_m+1}^{T} \frac{1}{(1+r)^{t-\theta_m}}}, \tag{A.2}$$

$$D_l(r) \equiv \frac{\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}}{\sum_{t=\theta_m+1}^{T} \frac{h}{(1+r)^{t-1}}} = \frac{\sum_{t=1}^{\theta_m} \frac{1}{(1+r)^{t-1}}}{\sum_{t=\theta_m+1}^{T} \frac{h}{(1+r)^{t-1}}} + \frac{1}{h} = \frac{\sum_{t=1}^{\theta_m} (1+r)^{\theta_m - t}}{h \sum_{t=\theta_m+1}^{T} \frac{1}{(1+r)^{t-\theta_m}}} + \frac{1}{h}. \tag{A.3}$$

Next, as discussed in the previous proof, $\gamma p_e(r)/w_h(r) = \gamma r^{\alpha_e/(1-\alpha_m)}$ increases with r. We also find that

$$\frac{w_l(r, p_n)}{w_h(r)} = p_n^{\frac{1}{1-\alpha_n}} r^{\frac{\alpha_m - \alpha_n}{(1-\alpha_m)(1-\alpha_n)}}$$

is increasing in r under our assumption that $\alpha_n < \alpha_m$.

A.3 Proof of Lemma 3

We show that

$$G_1(r, p_n) = \frac{\alpha_n}{1-\alpha_n}\omega_l(r, p_n)\bar{L}(r, p_n) + \frac{\alpha_m}{1-\alpha_m}\omega_h(r)\bar{H}(r, p_n) + \frac{\alpha_e - \alpha_m}{1-\alpha_m}\omega_h(r)^{1-\alpha_e}\gamma\bar{S}(r, p_n) - \bar{K}(r, p_n)$$

is decreasing in r if the conditions (i) and (iii) below hold, while it is decreasing in p_n if the conditions (ii) and (iii) hold.

(i) $\int_0^{\bar{a}_l(r,p_n)} \frac{\partial \bar{K}_l}{\partial r}(r,p_n)\,dF(a) + \int_{\bar{a}_l(r,p_n)}^{\bar{a}_h(r)} \frac{\partial \bar{K}_m}{\partial r}(a,r)\,dF(a) + \int_{\bar{a}_h(r)}^{\infty} \frac{\partial \bar{K}_h}{\partial r}(a,r)\,dF(a) \geq 0.$

(ii) $\frac{\partial \bar{K}}{\partial p_n}(r,p_n) > \frac{\alpha_n}{1-\alpha_n}\frac{\partial \omega_l}{\partial p_n}(r,p_n)\bar{L}(r,p_n) \equiv \frac{\alpha_n}{(1-\alpha_n)^2} p_n^{\frac{\alpha_n}{1-\alpha_n}} r^{-\frac{1}{1-\alpha_n}} \bar{L}(r,p_n).$

(iii) The income shares of capital satisfy $\alpha_n \leq \alpha_m \leq \alpha_e$.

On the one hand, the partial derivative of the excess demand for capital with respect to r is calculated as

$$
\begin{aligned}
\frac{\partial G_1}{\partial r}(r,p_n) = {} & \underbrace{\frac{\alpha_n}{1-\alpha_n}\frac{\partial \omega_l}{\partial r}(r,p_n)\bar{L}(r,p_n)}_{<0} + \underbrace{\frac{\alpha_m}{1-\alpha_m}\omega_h'(r)\bar{H}(r,p_n)}_{<0} + \underbrace{\frac{\alpha_e-\alpha_m}{1-\alpha_m}(1-\alpha_e)\omega_h(r)^{-\alpha_e}\omega_h'(r)\gamma\bar{S}(r,p_n)}_{\leq 0 \text{ if } \alpha_m \leq \alpha_e} \\
& + \underbrace{\frac{1}{r} f\big(\bar{a}_l(r,p_n)\big)\frac{\partial \bar{a}_l}{\partial r}(r,p_n)\left[\frac{\alpha_n}{1-\alpha_n} w_l(r,p_n)T - \frac{\alpha_m}{1-\alpha_m}\bar{a}_l(r,p_n)h_1 w_h(r)(T-\theta_m)\right]}_{<0 \text{ if } \alpha_n \leq \alpha_m, \text{ because } w_l T < a_l h_1 w_h (T-\theta_m)} \\
& + \underbrace{\frac{1}{r} f\big(\bar{a}_h(r)\big)\bar{a}_h'(r)\frac{\alpha_m}{1-\alpha_m}\bar{a}_h(r) w_h(r)\big[h_1(T-\theta_m) - h_2(T-\theta_h)\big]}_{<0 \text{ because } h_2 > [(T-\theta_m)/(T-\theta_h)]h_1} \\
& + \underbrace{\frac{\alpha_e-\alpha_m}{1-\alpha_m}\omega_h(r)^{1-\alpha_e}\gamma\Big[-\theta_m f\big(\bar{a}_l(r,p_n)\big)\frac{\partial \bar{a}_l}{\partial r}(r,p_n) - (\theta_h-\theta_m) f\big(\bar{a}_h(r)\big)\bar{a}_h'(r)\Big]}_{\leq 0 \text{ if } \alpha_m \leq \alpha_e} \\
& - \frac{\partial \bar{K}}{\partial r}(r,p_n).
\end{aligned}
$$

Thus, if $\frac{\partial \bar{K}}{\partial r}(r,p_n) \geq 0$, then $\frac{\partial G_1}{\partial r}(r,p_n) < 0$ under the condition that $\alpha_n \leq \alpha_m \leq \alpha_e$.

Now,

$$
\begin{aligned}
\frac{\partial \bar{K}}{\partial r}(r,p_n) = {} & \int_0^{\bar{a}_l(r,p_n)} \frac{\partial \bar{K}_l}{\partial r}(r,p_n)\,dF(a) + \int_{\bar{a}_l(r,p_n)}^{\bar{a}_h(r)} \frac{\partial \bar{K}_m}{\partial r}(r,p_n)\,dF(a) + \int_{\bar{a}_h(r)}^{\infty} \frac{\partial \bar{K}_h}{\partial r}(r,p_n)\,dF(a) \\
& + f\big(\bar{a}_l(r,p_n)\big)\frac{\partial \bar{a}_l}{\partial r}(r,p_n)\left[\bar{K}_l(r,p_n) - \bar{K}_m\big(\bar{a}_l(r,p_n),r\big)\right] \\
& + f\big(\bar{a}_h(r)\big)\bar{a}_h'(r)\left[\bar{K}_m\big(\bar{a}_h(r),r\big) - \bar{K}_h\big(\bar{a}_h(r),r\big)\right],
\end{aligned}
$$

where $\bar{K}_l(r,p_n) - \bar{K}_m(\bar{a}_l(r,p_n),r) = \sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\sum_{t=\tau}^{T-1}\left[\frac{w_l(r,p_n)}{(1+r)^{t-\tau}} - \frac{w_{m,t}(\bar{a}_l(r,p_n),\tau,r)}{(1+r)^{t-\tau}}\right] > 0,$

$$\text{because} \quad \sum_{t=\tau}^{\tau+T-1}\frac{w_l(r,p_n)}{(1+r)^{t-\tau}} = \sum_{t=\tau}^{\tau+T-1}\frac{w_{m,t}(\bar{a}_l(r,p_n),\tau,r)}{(1+r)^{t-\tau}} \quad \text{and}$$

$$\begin{cases} w_l(r,p_n) > w_{m,t}(\bar{a}_l(r,p_n),\tau,r) & t = \tau, \tau+1, ..., \tau+\theta_m-1, \\ w_l(r,p_n) < w_{m,t}(\bar{a}_l(r,p_n),\tau,r) & t = \tau+\theta_m, \tau+\theta_m+1, ..., \tau+T-1. \end{cases}$$

and $\bar{K}_m(\bar{a}_h(r),r) - \bar{K}_h(\bar{a}_h(r),r) = \sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\sum_{t=\tau}^{T-1}\left[\frac{w_{m,t}(\bar{a}_l(r,p_n),\tau,r)}{(1+r)^{t-\tau}} - \frac{w_{h,t}(\bar{a}_l(r,p_n),\tau,r)}{(1+r)^{t-\tau}}\right] > 0,$

$$\text{because} \quad \sum_{t=\tau}^{\tau+T-1}\frac{w_{m,t}(\bar{a}_h(r),\tau,r)}{(1+r)^{t-\tau}} = \sum_{t=\tau}^{\tau+T-1}\frac{w_{h,t}(\bar{a}_h(r),\tau,r)}{(1+r)^{t-\tau}} \quad \text{and}$$

$$\begin{cases} w_{m,t}(\bar{a}_h(r),\tau;r) = w_{h,t}(\bar{a}_h(r),\tau,r) & t = \tau, \tau+1, ..., \tau+\theta_m-1, \\ w_{m,t}(\bar{a}_h(r),\tau,r) > w_{h,t}(\bar{a}_h(r),\tau,r) & t = \tau+\theta_m, \tau+\theta_m+1, ..., \tau+\theta_h-1, \\ w_{m,t}(\bar{a}_h(r),\tau,r) < w_{h,t}(\bar{a}_h(r),\tau,r) & t = \tau+\theta_h, \tau+\theta_h+1, ..., \tau+T-1. \end{cases}$$

Therefore, when $\int_0^{\bar{a}_l(r,p_n)}\frac{\partial \bar{K}_l}{\partial r}(r,p_n)\,da + \int_{\bar{a}_l(r,p_n)}^{\bar{a}_h(r)}\frac{\partial \bar{K}_m}{\partial r}(a,r)\,da + \int_{\bar{a}_h(r)}^{\infty}\frac{\partial \bar{K}_h}{\partial r}(a,r)\,da \geq 0$, which is nothing but the condition (i), we have indeed $\frac{\partial \bar{K}}{\partial r}(r,p_n) \geq 0$.

On the other hand, the partial derivative of the excess demand for capital with respect to p_n is calculated as

$$\begin{aligned} \frac{\partial G_1}{\partial p_n}(r,p_n) = & \underbrace{\frac{\alpha_n}{1-\alpha_n}\frac{\partial \omega_l}{\partial p_n}(r,p_n)\bar{L}(r,p_n)}_{>0} \\ & + \underbrace{\frac{1}{r}f(\bar{a}_l(r,p_n))\frac{\partial \bar{a}_l}{\partial p_n}(r,p_n)\left[\frac{\alpha_n}{1-\alpha_n}w_l(r,p_n)T - \frac{\alpha_m}{1-\alpha_m}\bar{a}_l(r,p_n)h_1 w_h(r)(T-\theta_m)\right]}_{<0 \text{ if } \alpha_n \leq \alpha_m, \text{ because } w_l T < a_l h_1 w_h (T-\theta_m)} \\ & + \underbrace{\frac{\alpha_e-\alpha_m}{1-\alpha_m}\omega_h(r)^{1-\alpha_e}\gamma\left[-\theta_m f(\bar{a}_l(r,p_n))\frac{\partial \bar{a}_l}{\partial p_n}(r,p_n)\right]}_{\leq 0 \text{ if } \alpha_m \leq \alpha_e} \\ & - \frac{\partial \bar{K}}{\partial p_n}(r,p_n). \end{aligned}$$

The last term on the right-hand side can be calculated as

$$\frac{\partial \bar{K}}{\partial p_n}(r,p_n) = \int_0^{\bar{a}_l(r,p_n)}\frac{\partial \bar{K}_l}{\partial p_n}(r,p_n)\,dF(a) + f(\bar{a}_l(r,p_n))\frac{\partial \bar{a}_l}{\partial p_n}(r,p_n)\left[\bar{K}_l(r,p_n) - \bar{K}_m(\bar{a}_l(r,p_n),r)\right],$$

$$
\begin{aligned}
\text{where } \bar{K}_l(r,p_n) &= \sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\sum_{t=\tau}^{T-1}\left[\frac{w_l(r,p_n)}{(1+r)^{t-\tau}} - \frac{1}{(1+\rho)^{t-\tau}}\frac{\bar{I}_l(r,p_n)}{\Gamma_\tau(\tau)}\right] \\
&= \bar{I}_l(r,p_n)\sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\sum_{t=\tau}^{T-1}\left[\frac{1}{(1+r)^{t-\tau}}\frac{1}{\sum_{s=1}^{T}\frac{1}{(1+r)^{s-1}}} - \frac{1}{(1+\rho)^{t-\tau}}\frac{1}{\Gamma_\tau(\tau)}\right] \\
&= \bar{I}_l(r,p_n)\sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\left[\frac{\sum_{t=\tau}^{T-1}\frac{1}{(1+r)^{t-\tau}}}{\sum_{t=\tau}^{\tau+T-1}\frac{1}{(1+r)^{t-\tau}}} - \frac{\sum_{t=\tau}^{T-1}\frac{1}{(1+\rho)^{t-\tau}}}{\sum_{t=\tau}^{\tau+T-1}\frac{1}{(1+\rho)^{t-\tau}}}\right] \\
&= \bar{I}_l(r,p_n)\sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\left\{\left[1+\frac{\sum_{t=T}^{\tau+T-1}\frac{1}{(1+r)^{t-\tau}}}{\sum_{t=\tau}^{T-1}\frac{1}{(1+r)^{t-\tau}}}\right]^{-1} - \left[1+\frac{\sum_{t=T}^{\tau+T-1}\frac{1}{(1+\rho)^{t-\tau}}}{\sum_{t=\tau}^{T-1}\frac{1}{(1+\rho)^{t-\tau}}}\right]^{-1}\right\} \\
&= \bar{I}_l(r,p_n)\sum_{\tau=1}^{T-1}(1+r)^{T-1-\tau}\underbrace{\left\{\left[1+\frac{\sum_{t=T}^{\tau+T-1}\frac{1}{(1+r)^{t-\tau-(T-\tau)}}}{\sum_{t=\tau}^{T-1}(1+r)^{(T-\tau)-(t-\tau)}}\right]^{-1} - \left[1+\frac{\sum_{t=T}^{\tau+T-1}\frac{1}{(1+\rho)^{t-\tau-(T-\tau)}}}{\sum_{t=\tau}^{T-1}(1+\rho)^{(T-\tau)-(t-\tau)}}\right]^{-1}\right\}}_{>0,\quad \text{because } r>\rho \text{ in equilibrium with positive capital supply}}.
\end{aligned}
$$

Then, it follows from $\frac{\partial \bar{I}_l}{\partial p_n}(r,p_n) > 0$ that $\frac{\partial \bar{K}_l}{\partial p_n}(r,p_n) > 0$ (which we obtain from the above expression of $\bar{K}_l(r,p_n)$), and hence we have $\frac{\partial \bar{K}}{\partial p_n}(r,p_n) > 0$ from $\bar{K}_l(r,p_n) - \bar{K}_m\big(\bar{a}_l(r,p_n),r\big) > 0$.

Of course, this is not enough to obtain $\frac{\partial G_1}{\partial p_n}(r,p_n) < 0$. Going back to the expression of $\frac{\partial G_1}{\partial p_n}(r,p_n) < 0$, we see that if $\frac{\partial \bar{K}}{\partial p_n}(r,p_n) > \frac{\alpha_n}{1-\alpha_n}\frac{\partial \omega_l}{\partial p_n}(r,p_n)\bar{L}(r,p_n)$, which is nothing but the condition (ii), then $\frac{\partial G_1}{\partial p_n}(r,p_n) < 0$ under $\alpha_m \le \alpha_e$.

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